



## A TYPE II SUPERNOVAE CONSTRAINT ON $\nu_e - \nu_\mu$ MIXING \*

X. Shi and G. Sigl

Department of Astronomy & Astrophysics, The University of Chicago  
5640 S. Ellis Ave., Chicago IL 60637 USA  
and

NASA/Fermilab Astrophysics Center  
Box 500, Batavia, IL 60510 USA

### Abstract

The role of a resonant  $\nu_e - \nu_\mu$  oscillation is discussed in the event of a supernova explosion. It is concluded that a significant  $\nu_e - \nu_\mu$  mixing may hinder the ability of the supernova to explode. It may also cool the proto-neutron star too quickly with respect to the observed cooling time of several seconds. The constraints on the  $\nu_e - \nu_\mu$  mixing parameters based on the above arguments are calculated.

---

\* submitted to *Phys. Lett. B*



## 1. Introduction

Oscillations between active neutrinos  $\nu_\alpha$  and sterile neutrinos  $\nu_s$  have been discussed in various astrophysical and cosmological environments. Such examples include the oscillation between  $\nu_e$  and  $\nu_s$  as solutions to the solar neutrino problem [1], and oscillations between  $\nu_\alpha$  and  $\nu_s$  in Big Bang Nucleosynthesis which may affect the primordial  $^4\text{He}$  abundance [2-6]. Constraints on neutrino mixing parameters have been obtained by requiring the predicted  $^4\text{He}$  abundance under these neutrino mixings not to exceed the observed value [3-5]. Sterile neutrinos are also suggested as dark matter candidates if they have proper mixings with active neutrinos [6]. In this paper we attempt to consider  $\nu_e$ - $\nu_s$  mixing in another astrophysical environment—the Type II supernova (hereafter we omit “type II” for simplicity) core.

Since there isn't a fully convincing numerical supernova model at present, we try to construct our arguments as model-insensitively as possible. Supernova explosions occur when massive stars ( $> 8M_\odot$ ) reach the end of their evolution [7]. For such massive stars, an iron core is developed at the center of the progenitor star, surrounded sequentially by shells of silicon, oxygen, nitrogen, carbon, helium, and finally an envelope of hydrogen. Nuclear reactions occur at the interfaces of these shells and between the shell and the core. The initial iron core of the progenitor star has a density of about  $10^9 \text{ g/cm}^3$ , and is primarily supported by the pressure of the degenerate electron gas. As the nuclear burning of the silicon shell continues, more iron is deposited into the core, until it reaches the Chandrasekhar mass ( $\approx 1.4M_\odot$ ), when the core cannot be sufficiently supported by the electron degeneracy pressure and begins to collapse. In the meantime, due to the high density and the high temperature of the core, the deleptonization process  $e^- + p \rightarrow n + \nu_e$  occurs rapidly. When the density reaches  $10^{11} \text{ g/cm}^3$ , the mean free path of neutrinos  $l$  ( $\approx 10^3/\rho_{14} \text{ cm}$  for neutrinos of  $\sim 20 \text{ MeV}$ , where  $\rho_{14}$  is the core density divided by  $10^{14} \text{ g/cm}^3$ ) is shorter than the radius of the core  $R_{\text{core}} \sim 10\text{--}100\text{ km}$ . When the density is higher than  $10^{12} \text{ g/cm}^3$ , the diffusion timescale of neutrinos becomes longer than the timescale of the collapse of the core. Neutrinos are thereafter trapped inside the core instead of escaping freely. The collapse of the core finally stalls and bounces back when the center of the core reaches approximately nuclear density  $\text{few} \times 10^{14} \text{ g/cm}^3$ , at which the pressure of the degenerate neutron gas stops the gravitational collapse. The bounce creates a shock wave in the midst of the core that propagates outwards through the infalling matter. It is generally believed that if the core is small ( $< 1.4M_\odot$ ), the shock wave will have enough energy to blow the matter outside the core away and lead to an explosion event [8]. If the core is sufficiently massive, however, the shock wave quickly stalls within the core by losing energy in dissociating large quantities of infalling nuclei into free nucleons. It is currently

believed that in this case neutrinos deposit their energies into the shock several tenths of a second after the bounce and reheat the shock wave to eventually blow the outer layer away [9].

It is estimated that about 99% of the energy is emitted in the form of neutrino bursts. Therefore, the role of neutrinos in the supernova event is essential [9]. At the end of the collapse, electron neutrinos are trapped instead of streaming out of the core. The  $\nu_\mu$ ,  $\nu_\tau$ , and anti-neutrinos are also produced thermally inside the hot core with an average energy of several tens of MeV. This leads to a neutrino sphere with an average energy of  $\sim 20$  MeV trapped inside a dense core of nucleons and heavy nuclei. The number density of  $\bar{\nu}_e$  is highly suppressed at the collapsing stage of the supernova due to the existence of the degenerate  $\nu_e$  gas. Its number density builds up only after significant amount of  $\nu_e$  escapes from the core after deleptonization. It was shown by Wilson *et al.* that the  $\nu$  and  $\bar{\nu}$  annihilating at the neutrino sphere can deposit their energies in the shock and lift the shock front outwards, thus leading to an explosion [10]. Diffusive neutrinos further cool down the core (a proto-neutron star) to a neutron star after the explosion, on a timescale of about 10 seconds. This diffusive  $\bar{\nu}_e$  flux has been observed by Kamioka [11] and IMB [12] in conjunction with SN1987A, with a typical energy of 15–20 MeV and a timescale of about 10 seconds. Therefore, the current picture of a supernova event has received significant observational support.

## 2. Neutrino Mixing in the Supernova Core

The above picture assumes the standard electroweak theory. If neutrino mixing is present, the picture may change substantially. Many literatures have been focusing on the constraints on properties of neutrinos [13]. We will focus on the impact of a  $\nu_e$ - $\nu_s$  mixing on the supernova event, which hasn't been explored in full detail before [14]. In particular, instead of discussing the general cases of the mixing, we will discuss only the case of a resonant oscillation between  $\nu_e$  and  $\nu_s$ , which are relatively less model-dependent.

In the flavor basis the effective Hamiltonian of the  $\nu_\alpha$  (active)- $\nu_s$  (sterile) mixing system in a medium is given by [15]

$$H = \frac{1}{4E} \begin{pmatrix} -\delta m^2 \cos 2\theta + 4EV_{\nu_\alpha} & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}, \quad (1)$$

where  $E$  is the neutrino energy and  $\delta m^2 = m_2^2 - m_1^2$  is the difference of the squares of the two vacuum mass eigenvalues  $m_1$  and  $m_2$ . We assume  $\delta m^2 > 0$  when the sterile neutrino is heavier. Furthermore,  $\theta$  is the vacuum mixing angle and  $V_{\nu_\alpha}$  is the effective potential of the active neutrino  $\nu_\alpha$  in the medium. In a stellar environment [16],

$$\begin{aligned}
V_{\nu_e} &= \sqrt{2}G_F(N_e - 0.5N_n + 2N_{\nu_e} + N_{\nu_\mu} + N_{\nu_\tau}), \\
V_{\nu_\mu} &= \sqrt{2}G_F(-0.5N_n + N_{\nu_e} + 2N_{\nu_\mu} + N_{\nu_\tau})
\end{aligned} \tag{2}$$

where  $N_e$ ,  $N_n$  and  $N_{\nu_\alpha}$  are the number densities of electrons, neutrons, and neutrinos minus the number densities of their anti-particles, respectively.  $V_{\nu_\tau}$  is similar to  $V_{\nu_\mu}$  except that  $\nu_\mu$  and  $\nu_\tau$  are interchanged in the equation. The effective potential of a sterile neutrino is always zero. Based on  $CP$  symmetry,  $V_{\bar{\nu}_\alpha} = -V_{\nu_\alpha}$ . In the special situation of a supernova core,  $N_{\nu_\mu}$  and  $N_{\nu_\tau}$  are negligible, since  $\nu_\mu$  and  $\nu_\tau$  are only produced in  $\nu\bar{\nu}$  pairs via neutral current processes.

A resonance between the two mixing neutrinos occurs when the diagonal elements of the effective Hamiltonian eq. (1) become equal, i.e., when the neutrino energy equals the resonance energy  $E_r$  given by

$$E_r = \frac{\delta m^2 \cos 2\theta}{2V_{\nu_\alpha}}. \tag{3}$$

Inside a supernova core, where spherical symmetry is a reliable approximation, the region where such a resonance occurs for neutrinos with an energy  $E_r$  is a spherical shell with radius  $R_{\text{res}}(E_r)$ . It can be shown that if the sterile neutrinos resonantly converted from the active neutrinos escape freely from the core (we will discuss the condition for this later), the usual continuity equation relating the local rate of change of the active neutrino number density  $n_{\nu_\alpha}$  to the divergence of the corresponding local flux  $\vec{\phi}_{\nu_\alpha}$  is modified to

$$\dot{n}_{\nu_\alpha} = -\nabla \cdot \vec{\phi}_{\nu_\alpha} - \left[ \nabla E_r \cdot \frac{d\vec{\phi}_{\nu_\alpha}^{\text{out}}}{dE}(E_r) + \dot{E}_r \frac{dn_{\nu_\alpha}^{\text{out}}}{dE}(E_r) \right] P_{\nu_\alpha}. \tag{4}$$

Here,  $\vec{\phi}^{\text{out}}$  and  $n_{\nu_\alpha}^{\text{out}}$  are the partial flux and number density which only account for  $\nu_\alpha$ 's going outward with respect to the surface of constant resonance energy  $E_r$  (outward radial direction in our case). Furthermore,  $P_\alpha$  is the probability for an active neutrino going through the resonance to be converted to the sterile neutrino  $\nu_s$ . We have neglected terms corresponding to  $\nu_s \rightarrow \nu_\alpha$  conversions and  $\nu_\alpha \rightarrow \nu_s$  conversions of inward going  $\nu_\alpha$ , since we are only interested in the global  $\nu_\alpha$  loss by integrating eq. (4) over the whole SN core to which the sum of the neglected terms does not contribute.

In general,  $P_\alpha$  depends on the rate of decoherence of the neutrino flavor density matrix caused by non-forward scattering off the background [14] as well as on the adiabaticity of the resonance. In the limiting case where the width of the resonance region is much shorter than the mean free path of neutrinos, which is the case we will do our calculations,

$$P_\alpha(E_r) \approx 1 - \exp \left[ \frac{-\pi \delta m^2 \sin^2 2\theta}{4 \cos 2\theta} \left| \nabla E_r + \frac{\dot{E}_r}{c} \right|^{-1} \right]. \tag{5}$$

This is correct to the first order in the change of  $E_r$  in the resonance region [15].  $P_\alpha \sim 1$  corresponds to a highly adiabatic resonance whereas  $P_\alpha \ll 1$  represents a highly non-adiabatic resonance. On the other hand, when the width of the resonance region is much larger than the neutrino mean free path,  $P_\alpha \sim 0.5$  [14], which means the resonance is always “adiabatic”. Therefore, it can be seen from our following calculations that our constraints that exclude sufficiently adiabatic resonant mixings will equally apply to the case with large resonance width, as long as the  $\nu_s$  stream freely outside of the resonance region.

To get an order of magnitude estimate of the neutrino loss rate, we can further simplify eq. (4) by setting  $\nabla E_r \cdot (d\tilde{\phi}^{\text{out}}/dE)(E_r) \approx (dE_r/dr)ncf(E_r)E_r^2/\bar{E}^3$  and  $(dn^{\text{out}}/dE)(E_r) \approx nf(E_r)E_r^2/\bar{E}^3$  (we drop the subscript  $\nu_\alpha$  from now on). Here,  $c$  is the speed of light,  $f(E_r)$  is the occupation number at a state with  $E_r$ , and  $\bar{E}$  is the average neutrino energy which for the partially degenerate  $\nu_e$  is of the order of its chemical potential  $\mu$  and for other neutrino species (and all antineutrinos) is 3 times their temperatures in the supernova core. We can then integrate the second term on the r.h.s. of eq. (4) over the SN core to get the additional global  $\nu_\alpha$  loss rate  $\dot{N}_{\text{res}}$  and the additional global energy loss rate  $\dot{Q}_{\text{res}}$  due to the resonant conversion,

$$\begin{aligned}\dot{N}_{\text{res}} &\approx - \int_0^{R_{\text{core}}} 4\pi r^2 n \frac{dE_r}{dr} (c + v_r) f(E_r) \frac{E_r^2}{\bar{E}^3} P(E_r) dr, \\ \dot{Q}_{\text{res}} &\approx - \int_0^{R_{\text{core}}} 4\pi r^2 n \frac{dE_r}{dr} (c + v_r) f(E_r) \left(\frac{E_r}{\bar{E}}\right)^3 P(E_r) dr,\end{aligned}\quad (6)$$

where  $v_r = \dot{E}_r/|\nabla E_r|$ , the speed of the resonance shell for neutrinos with energy  $E_r$ . The integrals on the r.h.s. are dominated by integrands at  $E_r \sim \bar{E}$  if the resonance for neutrinos with energy  $\bar{E}$  ever occurs inside the core. When  $E_r \sim \bar{E}$ , we always expect  $|\dot{E}| \ll c \cdot dE/dr$  since otherwise the neutrinos will build up inside or dissipate from the core in  $R_{\text{core}}/c \sim 10^{-4}\text{s}$ , which contradicts our knowledge of supernovae. Therefore  $v_r$  and  $\dot{E}$  are neglected in the following discussion. If we denote  $N_{\text{tot}}$  and  $Q_{\text{tot}}$  as the total number and the total energy of  $\nu_\alpha$  inside the core, we can approximate eq. (6) by

$$\dot{N}_{\text{res}} \sim - \frac{cP(\bar{E})|_{\dot{E}_r=0}}{R_{\text{core}}} N_{\text{tot}}, \quad \dot{Q}_{\text{res}} \sim - \frac{cP(\bar{E})|_{\dot{E}_r=0}}{R_{\text{core}}} Q_{\text{tot}}. \quad (7)$$

One crucial quantity in the problem is the scale height of  $E_r$ ,  $|d \ln E_r / dr|^{-1}$ . A rough estimate for it is

$$\left| \frac{d \ln E_r}{dr} \right|^{-1} = \left| \frac{d \ln N_{\text{eff}}}{dr} \right|^{-1} \sim \left| \frac{d \ln n}{dr} \right|^{-1} \left| \frac{N_{\text{eff}}}{n} \right|, \quad (8)$$

where  $|d \ln n / dr|^{-1}$  is the scale height of the neutrino density  $n$  at the resonance region, and  $N_{\text{eff}} = V_{\nu_\alpha} / \sqrt{2} G_F$ . Without going into detailed models,

$$\left| \frac{d \ln n}{dr} \right|^{-1} > l. \quad (9)$$

In the following sections we apply these results to estimate the loss rates in the collapsing and the cooling phases of supernovae.

### 3. Constraints from the Collapsing Phase

During the collapsing phase of supernovae,  $\nu_e$  will encounter a resonance when the  $\nu_e$ - $\nu_s$  mixing satisfies

$$\delta m^2 \cos 2\theta = 2\sqrt{2}G_F E_r N_{\text{eff}} = 2\sqrt{2}G_F E_r (N_e - 0.5N_n + 2N_{\nu_e}). \quad (10)$$

Taking  $\bar{E} \sim 10\text{MeV}$ , and the density of the core  $\rho \sim 10^{12} \text{ g/cm}^3$  to several times  $10^{14} \text{ g/cm}^3$ , we find  $10^6 \text{ eV}^2 \lesssim \delta m^2 \lesssim 10^8 \text{ eV}^2$ . At this stage the  $\nu_e$ 's are already trapped in the SN core. The relevant time scale here is the free fall time scale of about  $10^{-3}\text{s} \sim R_{\text{core}}/(0.1c)$ . Then if  $\dot{N}_{\text{res}} > (0.1c/R_{\text{core}})N_{\text{tot}}$ ,  $\nu_e$  will never get trapped after the central density of the core reaches and exceeds the resonant value. The process is illustrated in Figure 1. It has been shown that no explosion occurs when the trapped lepton number is less than 0.9 times the lepton number before the collapse [17]. Therefore we require

$$P(\bar{E})|_{\dot{E}=0} \lesssim 0.1 \quad \text{for} \quad 10^6 \text{eV}^2 \lesssim \delta m^2 \lesssim 10^8 \text{eV}^2 \quad (11)$$

so that no serious leakage of  $\nu_e$  will occur. Because of eq. (5) this corresponds to

$$\frac{\delta m^2 \sin^2 2\theta}{\bar{E}} \left| \frac{d \ln E_r}{dr}(\bar{E}) \right|^{-1} \lesssim 0.1. \quad (12)$$

Since  $N_{\text{eff}} \sim n$ ,  $|d \ln E_r / dr(\bar{E})|^{-1} > l \sim 10^3 \text{cm}$  from eqs. (8) and (9). Taking  $\bar{E} \sim 10\text{MeV}$ , we conclude that  $\nu_e$ - $\nu_s$  mixings with

$$\delta m^2 \sin^2 2\theta \gtrsim 10^{-2} \text{eV}^2 \quad \text{and} \quad 10^6 \text{eV}^2 \lesssim \delta m^2 \lesssim 10^8 \text{eV}^2 \quad (13)$$

are ruled out so that the supernova core can trap enough lepton number to explode.

### 4. Constraints from the Cooling Phase

For  $\delta m^2 \lesssim 10^6 \text{ eV}^2$ , interesting constraints on the  $\nu_e$ - $\nu_s$  mixing come from the consideration of the resonance that occurs after the bounce. After the bounce,  $N_{\text{eff}} = N_e - 0.5N_n + 2N_{\nu_e}$  drops below zero very quickly at the outer part of the core [17,18]. (In general  $N_e = N_p \approx N_n$  before the collapse, where  $N_p$  is the number density of protons. After about 1/3 of protons are deleptonized into neutrons,  $N_{\text{eff}}$  will be negative.) However, at the center of the core, there is a significant build-up of  $\nu_e$ ,  $N_{\text{eff}}$  remains positive. Figure 2 shows roughly the relation between  $N_{\text{eff}}$  and the position inside the core at about 0.5 seconds after the bounce, as inferred from figure 10f of ref. 18 (similar relations exist for prompt-explosion models [17]). Thus, there exists a central part of the core with a positive

$V_{\nu_e}$  and an outer part of the core with a negative  $V_{\nu_e}$ . Furthermore, outside the core where no significant deleptonization has taken place,  $V_{\nu_e}$  is still positive. Therefore two resonance regions may form, (for both  $\nu_e$ - $\nu_s$  mixings and  $\bar{\nu}_e$ - $\bar{\nu}_s$  mixings), one inside the core and one outside the core, with

$$N_{\text{eff}} \approx 0, \quad (14)$$

if  $\delta m^2$  is much smaller than  $G_F E N_{\text{eff}}$  and can be neglected. If we take  $\bar{E} \sim 10$  MeV at the edge of the neutrino sphere, we can see from figure 2 that when  $\delta m^2 \lesssim 10^6 \text{ eV}^2$ , both  $\nu_e$  and  $\bar{\nu}_e$  can be resonantly converted into sterile neutrinos inside the core.

In this section we consider the resonance region inside the core. In this case, the scale height of  $E_r$  is much smaller than in the previous section due to eqs. (8) and (14). By using eq. (10) and  $n \lesssim \bar{E}^3/6\pi^2$ , we find for  $E_r \sim \bar{E}$ ,

$$\left| \frac{d \ln E_r}{dr} \right|^{-1}(\bar{E}) > l \left| \frac{N_{\text{eff}}}{n} \right| \gtrsim l \frac{3\pi^2 \delta m^2}{\sqrt{2} G_F \bar{E}^4}. \quad (15)$$

The concern is that if the resonance occurs with a sufficient adiabaticity, the intensive production of the freely streaming sterile neutrinos will cool down the core too quickly compared with the observed cooling time of about 10 seconds. This translates into

$$\frac{cP|_{\dot{E}_r=0}(\bar{E})}{R_{\text{core}}} \gg 1 \text{ sec}^{-1}, \quad \text{or equivalently} \quad P|_{\dot{E}_r=0}(\bar{E}) \gtrsim 10^{-3} \quad (16)$$

for  $R_{\text{core}} \sim 10$  to  $100 \text{ km}$ . By using an approximation

$$P|_{\dot{E}_r=0}(\bar{E}) \sim \frac{\delta m^2 \sin^2 2\theta}{\cos 2\theta} |\nabla E_r|^{-1} \quad \text{for } P \ll 1, \quad (17)$$

we conclude after combining eqs. (15), (16) and (17) that if

$$\left( \frac{\delta m^2 \sin^2 2\theta}{\cos 2\theta} \right) \left( \frac{3\pi^2 \delta m^2 l}{\sqrt{2} G_F \bar{E}^5} \right) \gtrsim 10^{-3}, \quad (18)$$

the proto-neutron star will cool down much faster than the observed timescale by losing neutrinos. Taking  $\bar{E} \sim 50$  MeV,  $l \sim 10^3 \text{ cm}$ , eq. (18) excludes  $\nu_e$ - $\nu_s$  mixings with

$$\delta m^2 \sin 2\theta \gtrsim 30 \text{ eV}^2 \quad \text{and} \quad \delta m^2 \lesssim 10^6 \text{ eV}^2. \quad (19)$$

## 5. Possible Tightening of the Bound by Reconversion

Since the effective potential  $V_{\nu_e}$  goes from negative at the edge of the core to positive outside the core, at least the  $\bar{\nu}_e$ - $\bar{\nu}_s$  will encounter a second resonance if  $\delta m^2 \lesssim 10^6 \text{ eV}^2$ . The density outside the core is quite low compared with that of the core. As a result, this outer resonance could be highly adiabatic since  $|d \ln E_r / dr|^{-1}$  could be much larger.

Then all the  $\bar{\nu}_s$  converted from  $\bar{\nu}_e$  deep in the core will be completely reconverted into  $\bar{\nu}_e$  which then freely escape from the supernova because the outer resonance is located above the neutrino sphere. But these neutrinos have energies  $\gtrsim 50$  MeV as they are originated from  $\bar{\nu}_e$ 's deep inside the neutrino sphere. Since the cross section of neutrinos in the  $\nu$ -e scattering experiments (Kamiokande and IMB) scales roughly with  $E^2$ , where  $E$  is the energy of the neutrinos, the fact that we didn't see any  $\bar{\nu}_e$  with  $E \gtrsim 50$  MeV in these experiments will put a tighter bound on  $\nu_e$ - $\nu_s$  mixings than that of the previous section. The average energy of  $\bar{\nu}_e$  observed by Kamiokande and IMB is  $\sim 20$  MeV [13]. Therefore, the above consideration could extend eq. (16) by two order of magnitude, because even if the energy loss rate due to the resonant conversion is only one tenth of the standard loss rate,  $\sim 5$  of the 20 neutrinos seen by IMB and Kamiokande should have had an energy of  $\gtrsim 50$  MeV. More generally, if in the future a neutrino detector detects  $\mathcal{N}$  supernova neutrinos with none of them having an energy  $\gtrsim 50$  MeV, the excluded parameter region would extend to

$$\delta m^2 \sin 2\theta \gtrsim \left(\frac{30}{\mathcal{N}}\right) \text{eV}^2 \quad \text{and} \quad \delta m^2 \lesssim 10^6 \text{eV}^2. \quad (20)$$

With the current data,  $\mathcal{N} = 20$ , the excluded region is

$$\delta m^2 \sin 2\theta \gtrsim 1 \text{eV}^2 \quad \text{and} \quad \delta m^2 \lesssim 10^6 \text{eV}^2. \quad (21)$$

One has to keep in mind, however, that eqs. (20) and (21) are only true if the outer resonance is highly adiabatic, which for eq. (21) requires the density scale height at the outer resonance to be larger than  $\sim (10/\sin 2\theta)\text{m}$ , which seems fairly reasonable when compared with the core dimensions.

## 6. Discussion and Summary

As we mentioned in the previous section, the above calculations are valid only if the sterile neutrinos have a mean free path larger than the radius of the core after passing the resonance. Outside the resonance region, the mean free path of sterile neutrinos is roughly  $l/\sin^2 \theta_m$  [3–5,14], where  $\theta_m$  is a typical medium mixing angle. In the case of the constraints from the collapsing phase discussed in section 3 this is of the same order as the vacuum mixing angle  $\theta$  whereas in the case of section 4 due to medium effects this is even suppressed compared to  $\theta$ . Our calculations are thus valid as long as

$$\sin^2 \theta < l/R_{\text{core}} \sim 10^{-3}. \quad (22)$$

Figure 3 shows the excluded region on the  $\nu_e$ - $\nu_s$  mixing parameter space from our consideration of a resonant  $\nu_e$ - $\nu_s$  mixing in both the pre-bounce and the post-bounce supernova core, eqs. (13) and (19), as well as the possible tighter bound eq. (21), and



eq. (22) under which the above bounds are valid. It also shows the parameter space excluded from Big Bang Nucleosynthesis, which gives a stricter constraints for  $\delta m^2 \lesssim 10^3$  eV<sup>2</sup> [3–5]. The hatched region in Fig. 3 shows the required mixing parameters for the sterile neutrino to contribute between 30% and 100% of the critical density of the universe [5,6] (assuming a Hubble constant between 50 km/sec/Mpc and 100 km/sec/Mpc). A significant part of the hatched region (at which the sterile neutrino serves as warm dark matter) is excluded by our constraints from the supernova.

In summary, we have considered the effect of a resonant  $\nu_e$ - $\nu_s$  mixing on a type II supernova. By requiring that a supernova retains enough leptons to explode and is not cooled down within a fraction of a second, we obtain constraints on the  $\nu_e$ - $\nu_s$  mixing parameter space. The excluded region includes the region required for the sterile neutrino to be the warm dark matter through  $\nu_e$ - $\nu_s$  oscillations in the early universe.

### Acknowledgement

We thank D. N. Schramm for highly valuable suggestions. This work was supported by DoE (Nuclear) and NSF at the University of Chicago, and by NASA through grant NAGW 2381 at Fermilab as well as the Alexander-von-Humboldt foundation.

## References:

1. D. N. Schramm and X. Shi, in *Proc. TAUP'93*, Gran Sasso Laboratory, Italy, 1993, ed. A. Bottino, Elsevier Science Publishers B.V. (to be published), and references therein.
2. A. Dolgov, *Sov. J. Nucl. Phys.*, **33**, 700 (1981); R. Barbieri and A. Dolgov, *Nucl. Phys.*, **B 349**, 743 (1991).
3. Enqvist *et al.*, *Nucl. Phys.* **B373**, 498 (1992).
4. J. M. Cline, *Phys. Rev. Lett.*, **68**, 3137 (1992).
5. X. Shi, D. N. Schramm and B. D. Fields, *Phys. Rev D.*, **48**, 2563 (1993).
6. Scott Dodelson and L. M. Widrow, FERMILAB-Pub-93/057-A.
7. For a review on supernovae, see *Supernovae*, ed. A. G. Petschek, Springer-Verlag, 1990; or *Supernovae, 10th Santa Cruz Summer Workshop in Astronomy and Astrophysics*, ed. S. E. Woosley, Springer-Verlag, 1989.
8. M. Takahara and K. Sato, *Prog. Theor. Phys.*, **71**, 524 and **72**, 978 (1984).
9. Section VII of *Supernovae, 10th Santa Cruz Summer Workshop in Astronomy and Astrophysics*, ed. S. E. Woosley, Springer-Verlag, 1989.
10. Ronald W. Mayle and James R. Wilson, in *Supernovae, 10th Santa Cruz Summer Workshop in Astronomy and Astrophysics*, ed. S. E. Woosley, Springer-Verlag, 1989.
11. K. Hirata *et al.*, *Phys. Rev. Lett.* **58**, 1490 (1987).
12. R. Bionta *et al.*, *Phys. Rev. Lett.* **58**, 1494 (1987).
13. D. N. Schramm and J. W. Truran, *Physics Report*, **189**, No. 2, 89 (1990) and references therein.
14. G. Raffelt and G. Sigl, *Astroparticle Physics* **1**, 165 (1993).
15. T. K. Kuo and James Panteleone, *Rev. Mod. Phys.*, **Vol. 61**, No. 4, 937 (1989).
16. D. Nötzold and G. Raffelt, *Nuclear Physics B* **307**, 924 (1988).
17. S. W. Bruenn, *Ap. J.* **340**, 955 (1989).
18. James R. Wilson and Ronald W. Mayle, in *The Nuclear Equation of State, Part A*, eds. W. Greiner and H. Stöcker, Plenum Press, New York, 1989.

**Figure Caption:**

Figure 1. An illustration of the configuration of  $V_{\nu_e}$  inside the core during the core collapse with (the dashed line) and without (the solid line) the  $\nu_e$ - $\nu_s$  mixing that satisfies eq. (13).

Figure 2. An illustration of  $N_{\text{eff}}$  inside the core at the bounce and several tenths of a second after the bounce.  $R_{ns}$  is the radius of the neutrino sphere.

Figure 3. On the  $\nu_e$ - $\nu_s$  mixing parameter space, the area enclosed by the solid line is excluded by our supernova consideration. The area enclosed by the long-dashed line is the possible extended bound due to the resonance outside the core. The area enclosed by short-dashed line is excluded by Big Bang Nucleosynthesis. The hatched region shows the parameters for a sterile neutrino dark matter candidate (generated a  $\nu_e$ - $\nu_s$  mixing in the early universe) to contribute 30% to 100% the critical density of the universe today (the Hubble constant is taken to be between 50–100km/sec/Mpc).

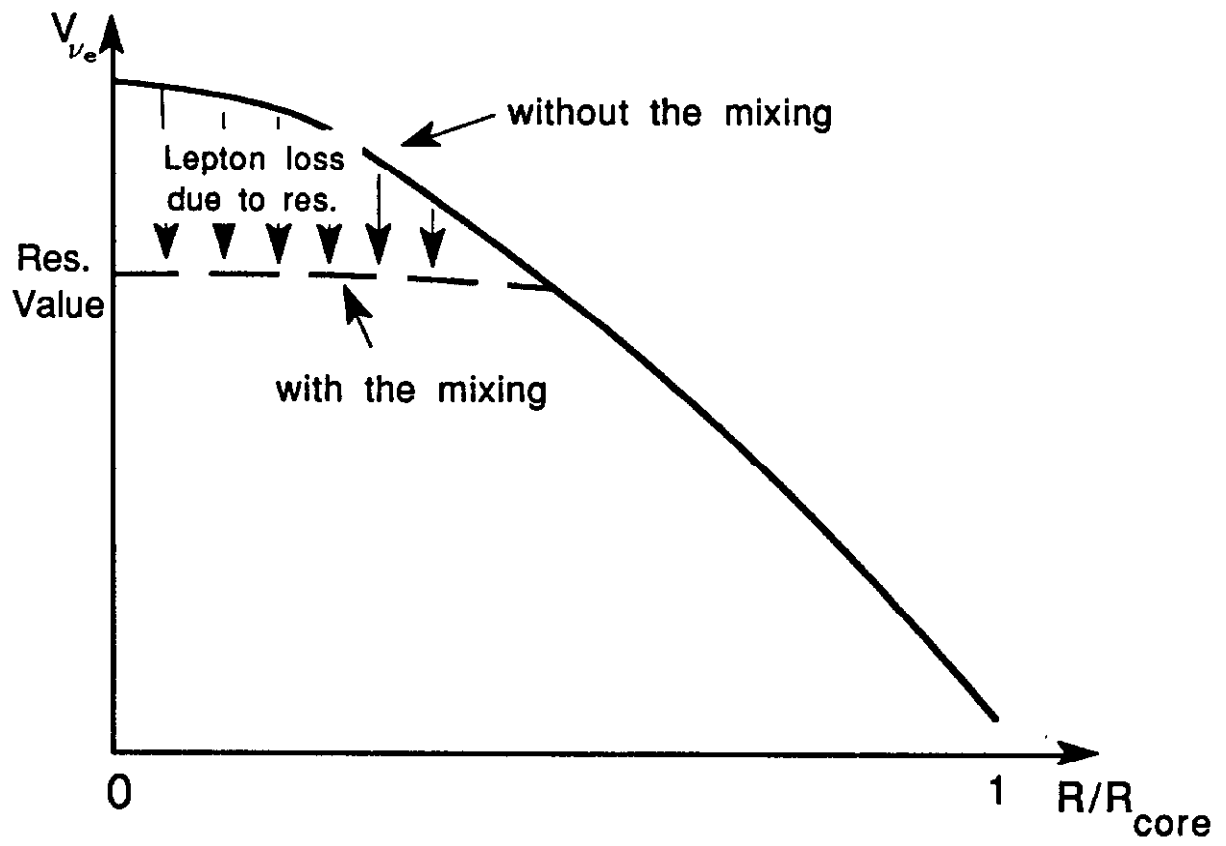


Fig. 1

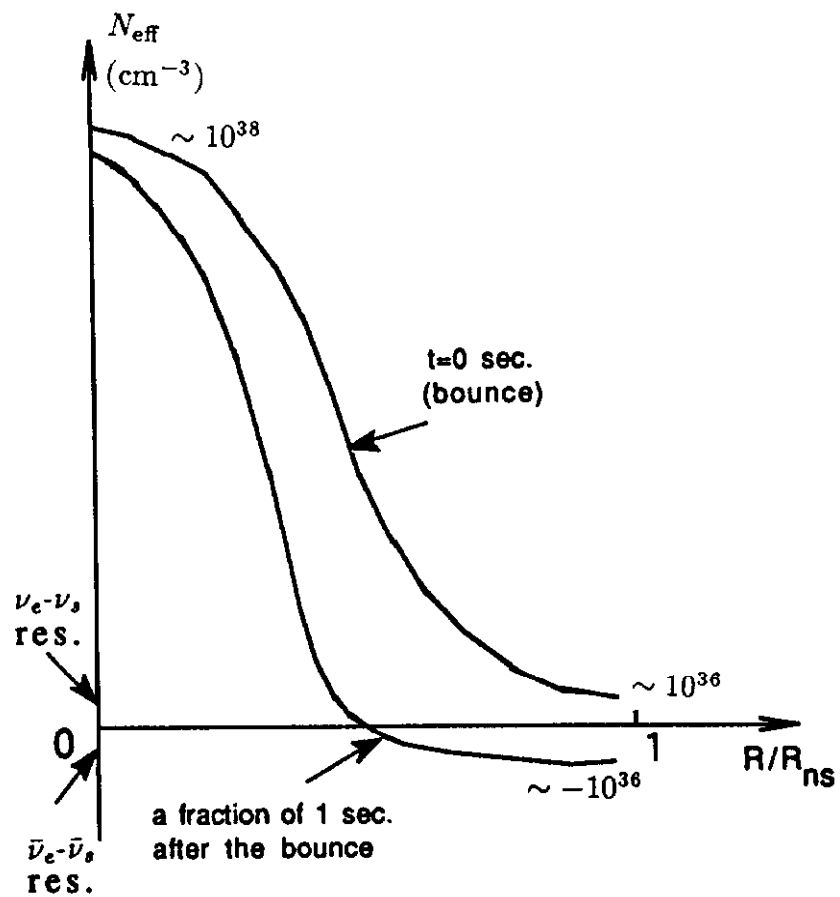


Fig. 2

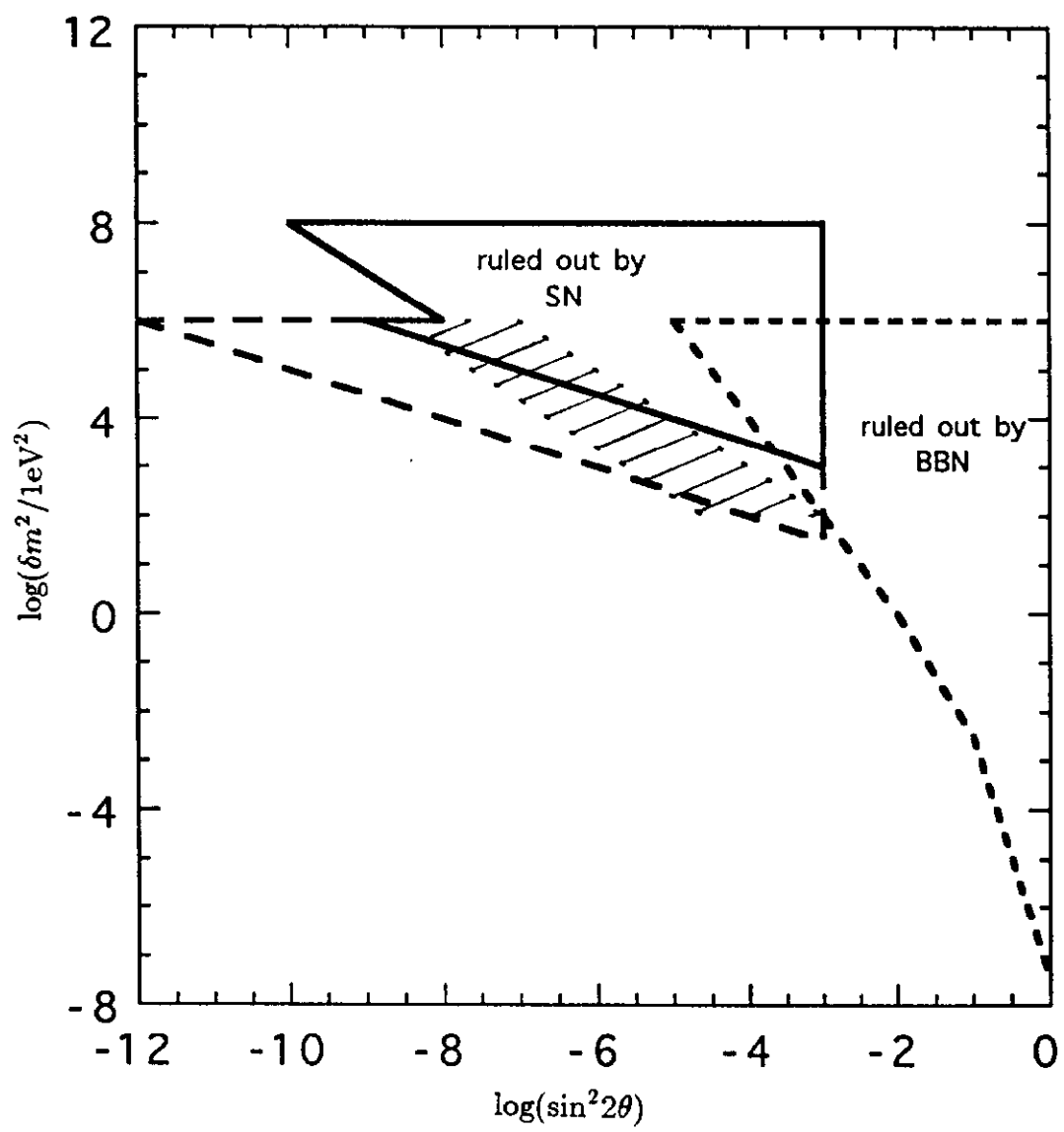


Fig. 3